Chapter 8 - Slutsky Equation

→ We know that when prices change, the quantity demanded changes.

→ It's often useful to decompose this change in demand into the part that is due to a change in relative prices and that which is due to a change in purchasing power

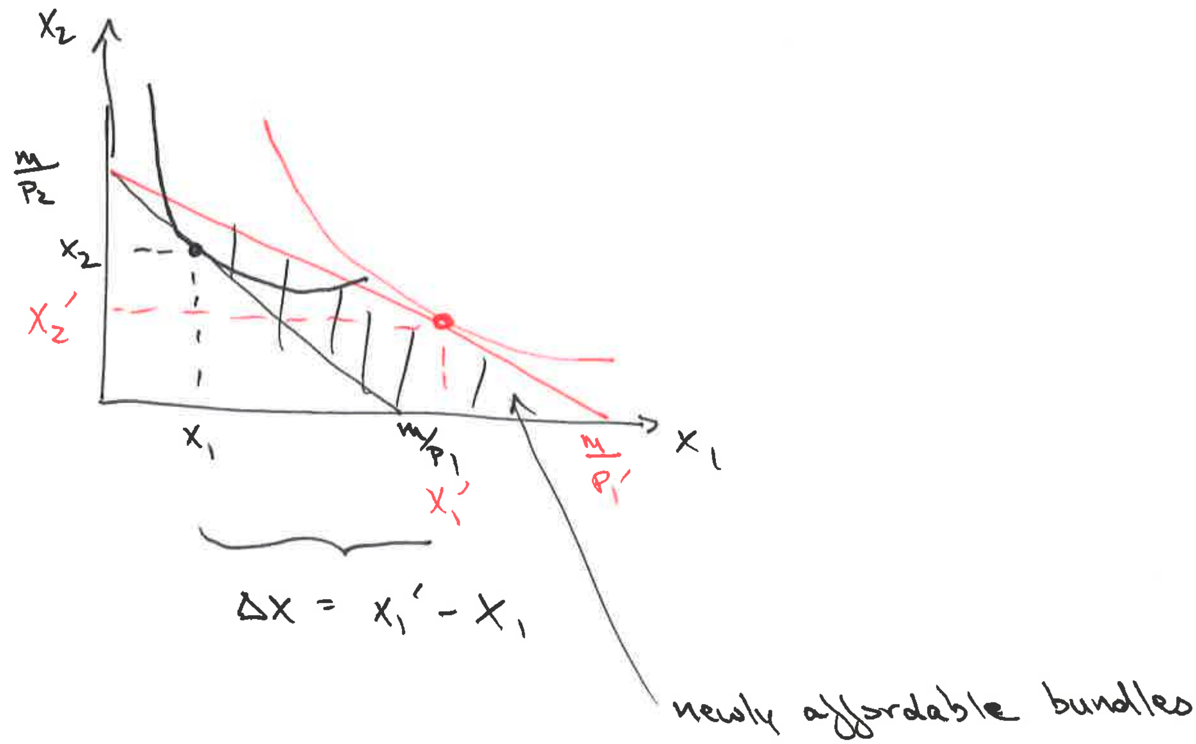
→ We'll call these two components of the overall change in demand the substitution effect and the income effect, respectively.

→ The Slutsky Equation (or Slutsky identity) just says:

Total Change in demand = change in demand due to subs. effect + change in demand due to the income effect

Substitution Effects

→ consider a change in prices. e.g. p_1 goes down to p_1':



→ so you can see that a change in prices does 2 things:

1) It changes the slope of the BL

2) It changes the set of affordable bundles

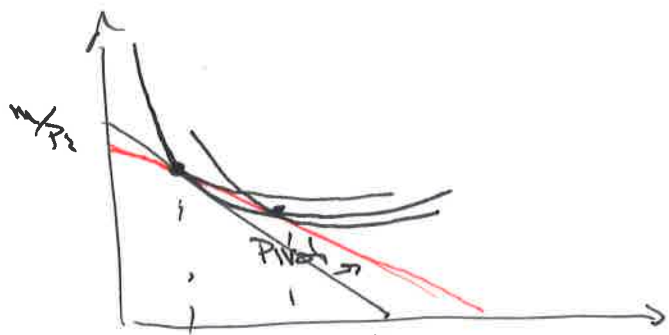
→ These 2 changes correspond to the substitution and income effects.

→ To decompose the total change into these 2 effects, consider the change in price affecting the BL in 2 discrete steps:

→ Step 1: The budget line pivots about the bundle that was the original choice.

→ That is the BL still goes through (x_1, x_2), but its slope changes from

$$-\frac{p_1}{p_2} \text{ to } -\frac{p_1'}{p_2}$$



$\underbrace{x_1 \rightarrow x_1^s}$

subs effect

→ the subs effect is then the difference btwn the original bundle and that demanded ~~in~~ under this pivoted BL

→ Now consider this same thing through equations.

→ The key here is to note that we will be adjusting purchasing power so that the original bundle is just affordable under the ~~x~~ new prices.

w/ the original bundle, we had

$$p_1 x_1 + p_2 x_2 = m$$

w/ the change in price we want to adjust income to m' such that:

$$\underbrace{p_1' x_1 + p_2 x_2}_{\text{expend at new prices}} = \underbrace{m'}_{\text{adjusted income}}$$

→ subtracting the first from the second we can find the adjustment to income that's needed to keep the bundle affordable:

$$\begin{array}{rl} m' &= p_1' x_1 + \cancel{p_2 x_2} \\ -\ m &= p_1 x_1 + \cancel{p_2 x_2} \\ \hline \end{array}$$

$$\underbrace{m' - m}_{\Delta m} = x_1 \underbrace{(p_1' - p_1)}_{\Delta p_1}$$

$$\Delta m = x_1 \, \Delta p_1$$

→ so we know how we have to change income as a function of the original demand and the price change.

→ so to find the substitution effect (or change in compensated demand), we'll ~~the~~ take the demand function for

$x_1 = x_1(p_1, p_2, m)$ and solve:

$$\underbrace{\Delta x_1^s}_{\substack{\text{change in} \\ x_1 \text{ due to} \\ \text{subs effect}}} = \underbrace{x_1(p_1', p_2, m')}_{\substack{\text{demand at} \\ \text{new price} \\ \text{and adj.} \\ \text{income}}} - \underbrace{x_1 \cancel{(p_1, m)} (p_1, p_2, m)}_{\text{original demand}}$$

Example of finding the substitution effect:

let demand for thin mints be given

by: $x_1 = \frac{m}{4p_1}$

→ originally income is \$10 and the price of thin mints \$2 per box

→ so original quantity demanded is given as:

$$x_1 = \frac{m}{4p_1} = \frac{10}{4 \times 2} = \frac{10}{8} = \frac{5}{4} = 1.25 \text{ boxes}$$

→ now suppose the price of a box increases to \$2.5

→ To keep the original bundle affordable, we need

$$\Delta m = x_1 \Delta p = x_1(2.5 - 2) = 1.25(0.5)$$
$$= 0.625$$

→ which means adjusted income is given by:

$$m' = m + \Delta m = 10 + 0.625 = 10.625$$

→ so to find the subs. effect we do:

$$\Delta X^S = x_1(p_1', m') - x_1(p_1, m)$$

$$= \frac{10.625}{4 \times 2.5} - 1.25$$

$$= \frac{10.625}{10} - 1.25$$

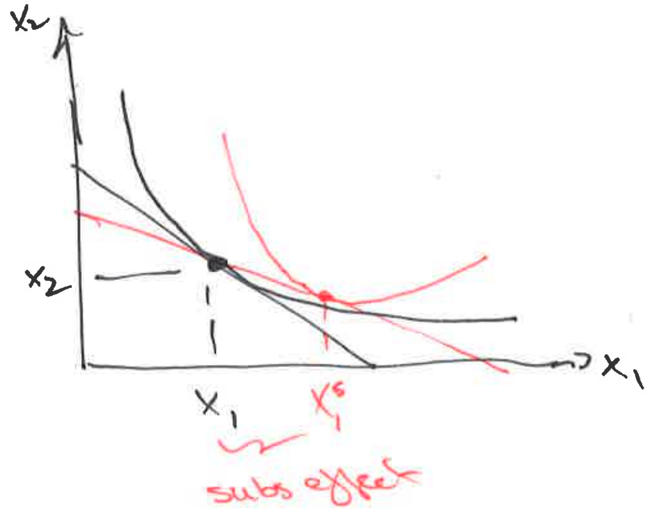
$$= 1.0625 - 1.25 = \underline{-0.1875}$$

<u>The Income Effect</u>

→ This is the change in demand that is the result of a change in what is affordable, holding ~~price~~ the price ratio constant

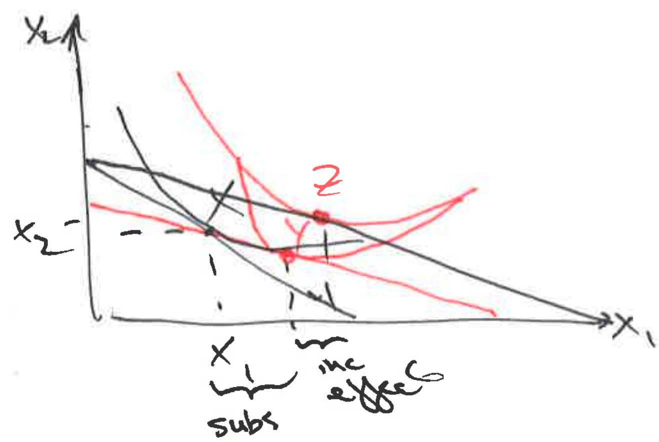
→ Recall how we were breaking the ~~price~~ total change ~~into~~ in demand into the two components, ~~we due to~~ by considering the changes to the BL in 2 discrete steps

→ The first step was the pivot of the BL about the original bundle:



→ The second step is to then shift the BL in or out from this pivoted line. This will then represent the BL under new prices with original income!

→ e.g.



→ we'll then be able to find the income effect as the difference between the bundle demanded under the "pivoted BL" and that demanded after the shift in the BL.

→ Algebraically, we have:

$$\underbrace{\Delta x_1^n}_{\substack{\text{change in} \\ x_1 \text{ due to} \\ \text{the income} \\ \text{effect}}} = \underbrace{x_1(p_1', p_2, m)}_{\substack{\text{demand} \\ \text{under new} \\ \text{price and} \\ \text{org. income}}} - \underbrace{x_1(p_1', p_2, m')}_{\substack{\text{demand under} \\ \text{new price and} \\ \text{adjusted income}}}$$

→ Example of finding income effect:

→ Take cookie example:

$$x_1 = \frac{m}{4p_1}$$

$$p_1 = \$2.$$

$$p_1' = \$2.5$$

→ we already found $x_1(p_1', p_2, m') = 1.0625$

→ To find $x_1(p_1', p_2, m)$ we have

$$x_1 = \frac{m}{4 \times p_1'} = \frac{10}{4 \times 2.5} = \frac{10}{10} = 1$$

→ so income effect is:

$$\Delta x_1^n = x_1(p_1', p_2, m) - x_1(p_1', p_2, m')$$

$$= 1 - 1.0625$$

$$= -0.0625$$

→ Thus the income effect is -0.0625
and the subs effect is -0.1875

The total change in demand

→ The total change in demand is the diff btwn demand at the original price and that under the new price (income and credit change).

Example: cookies

$$\Delta x_1 = x_1(p_1', p_2, m) - x_1(p_1, p_2, m)$$

$$= \frac{10}{4 \times 2.5} - \frac{10}{4 \times 2}$$

$$= \frac{10}{10} - \frac{10}{8}$$

$$= 1 - 1.25 = -0.25$$

How does this compare to the income and subs effects?

$$\Delta x^s = -0.1875$$
$$\Delta x^n = -0.0625$$
$$\overline{\quad -0.25 \quad}$$

→ the sum of the 2 effects equals the total effect.

→ Is it always the case that the sum of the income and subs effect equals the total change in demand?

→ consider the sum:

$$\Delta x_1^s + \Delta x_1^n = \underbrace{\left(x_1(\cancel{p_1', p_2}, m') - x_1(p_1, p_2, m)\right)}_{\Delta x^s}$$

$$+ \left(x_1(p_1', p_2, m) - x_1(\cancel{p_1', p_2}, m')\right)$$

$$= \underbrace{x_1(p_1', p_2, m) - x_1(p_1, p_2, m)}_{\Delta x_1}$$

→ so we can prove

$$\Delta x_1 = \Delta x_1^s + \Delta x_1^n$$

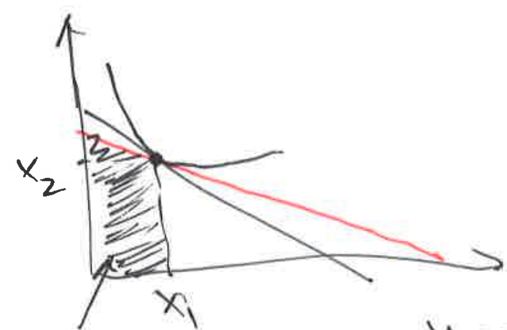
→ the total change is the sum of the income and subs. effects.

→ This equation: $\Delta x_1 = \Delta x_1^s + \Delta x_1^n$ is called the <u>Slutsky identity</u>

What can we say about the sign of the total change ~~for~~ in demand for a change in prices?

→ Substitution effect: the sign of Δx^s must be negative

→ why? use revealed preference:



Points w/ x_1 less than original x_1 all affordable before line pivoted, but not chosen

→ implies want prefers (x_1, x_2) to and newly ~~afforda~~ affordable bundle w/ less x_1

$$\Rightarrow x_1(p_1', m') \geq x_1(p_1, m)$$

→ so the subs effect is always negative.

→ The income effect: ambiguous

→ if normal good, then $\Delta x^n < 0$

→ if inferior good, then $\Delta x^n > 0$

why? Price ↑ is like income falling. When income falls, demand less normal goods, but more inferior goods

→ So, sign of total ~~per~~ change in demand depends on whether the good is normal or inferior

1) If a normal good, then Δx^s and Δx^n move in same direction and we know the sign of the total change:

$$\underset{(-)}{\Delta x} = \underbrace{\underset{(-)}{\Delta x^s} + \underset{(-)}{\Delta x^n}}_{\text{subs and income effects both negative}}$$

⇒ If the good is an inferior good, then Δx^s and Δx^n move in opposite directions and so the sign of the total change in the demand is ambiguous - it depends on the magnitude of the income effect compared to the subs. effect:

$$\underset{(?)}{\Delta x_1} = \underset{(-)}{\Delta x_1^s} + \underset{(+)}{\Delta x_1^n}$$

→ If Δx_1^n large, so that $\Delta x_1 > 0$, then we have the case of a Giffen good - where demand increases as prices increase

→ Thus a Giffen good must be an ~~g.~~ inferior good

→ But an inferior good need not be a Giffen good

A Note on Substitution Effects

→ We found the substitution effect by holding purchasing power constant

→ This is sometimes called the Slutsky substitution effect.

→ One could ~~also~~ hold utility constant, rather than income constant

→ if one does this, the resulting substitution effect is called the Hicks substitution effect

→ This concept is often useful for thinking about how a consumers welfare changes as a result of price changes.

→ We prob. won't do much w/ this here, but you should be aware.